Lambda Calculus and Formal Grammar
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Reasoning about contexts in Lambek Grammars

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Lambda in the syntax

de Groote 2001a:

An asymmetry in Lambek grammars:

syntax: *everyone*: $\mathrm{S}/(\mathrm{DP}\backslash\mathrm{S})$

semantics: $\lambda P \forall x.Px : \langle\langle \mathtt{e}, \mathtt{t}\rangle, \mathtt{t}\rangle$

"This asymmetry can be broken by ... allowing λ-terms on the syntactic side."

Muskens 2003:

Word order and constituency should be dealt with on a separate level. I will discuss a grammatical formalism (Lambda Grammars) that allows one to combine signs that are sequences of λ-terms with the help of linear combinators (essentially closed pure λ-terms in which each abstractor binds exactly one variable).

Muskens 2001: Oehrle, Cresswell, Curry...

More on ACGs later...

Plan

- NL_{CL}: Non-associative Lambek grammar with combinators
  - CL = Combinatory Logic, Context Logic, Continuation Logic
- Soundness and completeness for NL_{CL}
  - Conservative wrt NL
- NL_{λ}: NL with lambdas (for now, a notational variant of NL_{CL})
  - Lambdek Grammar
- Examples, comparisons, interpretations
  - Extends Barker and Shan 2006 to nested contexts (related to Morrill et al.'s Discontinuous Lambek Grammar)
    - Quantificational binding
    - Scope-taking adjectives: *same*
  - Undelimited continuations (Moortgat et al.)? No, delimited.
  - Approximating ACGs in a Lambek grammar.

$\mathbf{NL}_{CL}$

$\mathbf{NL}_{CL}$: a two-mode, non-associative Lambek grammar

- A set of atomic formula symbols $\mathcal{A} = \{\text{DP}, \text{S}, ...\}$
- Two modes, default ($\backslash$, $/$) and continuation ($\mathbin{\backslash\!\!\backslash}$, $/\!\!/$)
- A set of formulas $\mathcal{F} \supset \mathcal{A}$ such that for all $A, B \in \mathcal{F}$

$$\mathcal{F} ::= A\backslash B \quad | \quad B/A \quad | \quad A\mathbin{\backslash\!\!\backslash} B \quad | \quad B/\!\!/A$$

- A set of structures $\mathcal{S} \supset \mathcal{F}$ such that for all $X, Y \in \mathcal{S}$

$$\mathcal{S} ::= X \bullet Y \quad | \quad X \circ Y \quad | \quad \mathsf{I} \quad | \quad \mathsf{B} \quad | \quad \mathsf{C}$$

- The usual logical rules (next slide)
- Three structural postulates (next slide after).
- Note: fusion formulas have been omitted for simplicity. They can be conservatively added to the logic in the natural way (Restall theorem 11.52). I have set the symbols $\bullet$ and $\circ$ in red to emphasize that they are structural connectives (KJB).

Logical rules (sequent presentation)

$$\frac{\Gamma \vdash A \qquad \Sigma[B] \vdash C}{\Sigma[(\Gamma \bullet A \backslash B)] \vdash C} \backslash L \qquad \frac{A \bullet \Gamma \vdash C}{\Gamma \vdash A \backslash C} \backslash R$$

$$\frac{\Gamma \vdash A \qquad \Sigma[B] \vdash C}{\Sigma[(B / A \bullet \Gamma)] \vdash C} / L \qquad \frac{\Gamma \bullet B \vdash C}{\Gamma \vdash C / B} / R$$

$$\frac{\Gamma \vdash A \qquad \Sigma[B] \vdash C}{\Sigma[(\Gamma \circ A \mathbin{\backslash\!\!\backslash} B)] \vdash C} \mathbin{\backslash\!\!\backslash} L \qquad \frac{A \circ \Gamma \vdash C}{\Gamma \vdash A \mathbin{\backslash\!\!\backslash} C} \mathbin{\backslash\!\!\backslash} R$$

$$\frac{\Gamma \vdash A \qquad \Sigma[B] \vdash C}{\Sigma[(B \mathbin{/\!\!/} A \circ \Gamma)] \vdash C} \mathbin{/\!\!/} L \qquad \frac{\Gamma \circ B \vdash C}{\Gamma \vdash C \mathbin{/\!\!/} B} \mathbin{/\!\!/} R$$

Perfectly ordinary.

Structural rules for $\mathbf{NL}_{CL}$:

Restall:

> [I (Restall's '0')] is "a zero-place punctuation mark" (p. 30), where punctuation marks "stand to structures in the same way that connectives stand to fomulae" (p. 19).

$$\frac{p}{p \circ \mathsf{I}}\ \mathsf{I}$$

$$\frac{p \bullet (q \circ r)}{q \circ ((\mathsf{B} \bullet p) \bullet r)}\ \mathsf{B}$$

$$\frac{(p \circ q) \bullet r}{p \circ ((\mathsf{C} \bullet q) \bullet r)}\ \mathsf{C}$$

- I is a right identity with respect to $\circ$
- B governs mixed commutativity involving $\bullet$ and $\circ$
- C governs mixed associativity involving $\bullet$ and $\circ$
- Other interpretations later

Example derivation of *John saw everyone*

$$\vdots$$

$$\dfrac{\mathrm{DP} \bullet ((\mathrm{DP}\backslash \mathrm{S})/\mathrm{DP} \bullet \mathrm{DP}) \vdash \mathrm{S}}{\textit{John} \bullet (\textit{saw} \bullet \mathrm{DP}) \vdash \mathrm{S}}\ \mathrm{LEX}$$

$$\dfrac{\textit{John} \bullet (\textit{saw} \bullet \mathrm{DP}) \vdash \mathrm{S}}{\textit{John} \bullet (\textit{saw} \bullet (\mathrm{DP} \circ \mathsf{I})) \vdash \mathrm{S}}\ \mathsf{I}$$

$$\dfrac{\textit{John} \bullet (\textit{saw} \bullet (\mathrm{DP} \circ \mathsf{I})) \vdash \mathrm{S}}{\textit{John} \bullet (\mathrm{DP} \circ ((\mathsf{B} \bullet \textit{saw}) \bullet \mathsf{I})) \vdash \mathrm{S}}\ \mathsf{B}$$

$$\dfrac{\textit{John} \bullet (\mathrm{DP} \circ ((\mathsf{B} \bullet \textit{saw}) \bullet \mathsf{I})) \vdash \mathrm{S}}{\mathrm{DP} \circ ((\mathsf{B} \bullet \textit{John}) \bullet ((\mathsf{B} \bullet \textit{saw}) \bullet \mathsf{I})) \vdash \mathrm{S}}\ \mathsf{B}$$

$$\dfrac{\mathrm{DP} \circ ((\mathsf{B} \bullet \textit{John}) \bullet ((\mathsf{B} \bullet \textit{saw}) \bullet \mathsf{I})) \vdash \mathrm{S}}{(\mathsf{B} \bullet \textit{John}) \bullet ((\mathsf{B} \bullet \textit{saw}) \bullet \mathsf{I}) \vdash \mathrm{DP}\backslash\!\!\backslash \mathrm{S}}\ \backslash\!\!\backslash R$$

$$\dfrac{(\mathsf{B} \bullet \textit{John}) \bullet ((\mathsf{B} \bullet \textit{saw}) \bullet \mathsf{I}) \vdash \mathrm{DP}\backslash\!\!\backslash \mathrm{S} \qquad \mathrm{S} \vdash \mathrm{S}}{\mathrm{S}/\!\!/(\mathrm{DP}\backslash\!\!\backslash \mathrm{S}) \circ ((\mathsf{B} \bullet \textit{John}) \bullet ((\mathsf{B} \bullet \textit{saw}) \bullet \mathsf{I})) \vdash \mathrm{S}}\ /\!\!/ L$$

$$\dfrac{\mathrm{S}/\!\!/(\mathrm{DP}\backslash\!\!\backslash \mathrm{S}) \circ ((\mathsf{B} \bullet \textit{John}) \bullet ((\mathsf{B} \bullet \textit{saw}) \bullet \mathsf{I})) \vdash \mathrm{S}}{\textit{everyone} \circ ((\mathsf{B} \bullet \textit{John}) \bullet ((\mathsf{B} \bullet \textit{saw}) \bullet \mathsf{I})) \vdash \mathrm{S}}\ \mathrm{LEX}$$

$$\dfrac{\textit{everyone} \circ ((\mathsf{B} \bullet \textit{John}) \bullet ((\mathsf{B} \bullet \textit{saw}) \bullet \mathsf{I})) \vdash \mathrm{S}}{\textit{John} \bullet (\textit{everyone} \circ ((\mathsf{B} \bullet \textit{saw}) \bullet \mathsf{I})) \vdash \mathrm{S}}\ \mathsf{B}$$

$$\dfrac{\textit{John} \bullet (\textit{everyone} \circ ((\mathsf{B} \bullet \textit{saw}) \bullet \mathsf{I})) \vdash \mathrm{S}}{\textit{John} \bullet (\textit{saw} \bullet (\textit{everyone} \circ \mathsf{I})) \vdash \mathrm{S}}\ \mathsf{B}$$

$$\dfrac{\textit{John} \bullet (\textit{saw} \bullet (\textit{everyone} \circ \mathsf{I})) \vdash \mathrm{S}}{\textit{John} \bullet (\textit{saw} \bullet \textit{everyone}) \vdash \mathrm{S}}\ \mathsf{I}$$

$$\mathbf{everyone}(\lambda x.\mathbf{saw}\, x\, \mathbf{j})$$

- Completely ordinary Curry-Howard labeling.
- The two usual classes of derivations for *Someone saw everyone*.
- Long distance scope-taking (*Someone asked everyone to leave*).

More interesting derivations later.

Soundness and completeness for $\mathbf{NL}_{CL}$

Frames for NL$_{CL}$

A frame $\mathcal{F}$ consists of

- A (flat) set of points $\mathcal{P}$
- 3-place accessibility relations $R_\bullet$ and $R_\circ$
- 1-place predicates I, B, and C

Models

A model $\mathfrak{M}$ for NL$_{CL}$ is a frame along with an evaluation relation $\Vdash$ that satisfies the following:

Evaluation

$$x \Vdash B/A \text{ iff } \forall y, z.(R_\bullet xyz \wedge y \Vdash A) \rightarrow z \Vdash B$$

$$y \Vdash A\backslash B \text{ iff } \forall x, z.(R_\bullet xyz \wedge x \Vdash A) \rightarrow z \Vdash B$$

$$(z \Vdash A \bullet B \text{ iff } \exists x, y.R_\bullet xyz \wedge x \Vdash A \wedge y \Vdash B)$$

$$x \Vdash B /\!\!/ A \text{ iff } \forall y, z.(R_\circ xyz \wedge y \Vdash A) \rightarrow z \Vdash B$$

$$y \Vdash A \backslash\!\!\backslash B \text{ iff } \forall x, z.(R_\circ xyz \wedge x \Vdash A) \rightarrow z \Vdash B$$

$$(z \Vdash A \circ B \text{ iff } \exists x, y.R_\circ xyz \wedge x \Vdash A \wedge y \Vdash B)$$

$$x \Vdash \mathsf{I} \text{ iff } x \in I$$

$$x \Vdash \mathsf{B} \text{ iff } x \in B$$

$$x \Vdash \mathsf{C} \text{ iff } x \in C$$

$$z \Vdash p \bullet q \text{ iff } \exists x, y.R_\bullet xyz \wedge x \Vdash p \wedge y \Vdash q$$

$$z \Vdash p \circ q \text{ iff } \exists x, y.R_\circ xyz \wedge x \Vdash p \wedge y \Vdash q$$

Frame conditions

For structural postulate P, construct $F(P)$ as follows (Restall 249):

Propositional variables: $F(p) = (p = x)$

Structural connectives, either

zero-place: $F(\mathsf{I}) = Ix$

$F(\mathsf{B}) = Bx$

$F(\mathsf{C}) = Cx$

Two-place: $F(X \bullet Y) = \exists y \exists z. R_\bullet yzx \wedge F(X)[x := y] \wedge F(Y)[x := z]$

$F(X \circ Y) = \exists y \exists z. R_\circ yzx \wedge F(X)[x := y] \wedge F(Y)[x := z]$

Then for a structural rule $P = \dfrac{\Sigma[\Gamma] \vdash A}{\Sigma[\Gamma'] \vdash A}$ in which Γ and Γ' contain $p_1, p_2, ..., p_n$ as propositional variables,

$$F(P) = \forall x, p_1, p_2, ..., p_n . F(\Gamma') \rightarrow F(\Gamma)$$

Example: right identity

$$F(\frac{p}{p \circ \mathsf{I}}) =$$

$$\forall xp.(\exists y \exists z R_\circ yzx \wedge F(p)[x := y] \wedge F(\mathsf{I})[x := z]) \longrightarrow (p = x)$$
$$\forall xp.(\exists y \exists z R_\circ yzx \wedge (p = x)[x := y] \wedge F(\mathsf{I})[x := z]) \longrightarrow (p = x)$$
$$\forall xp.(\exists y \exists z R_\circ yzx \wedge (p = y) \wedge F(\mathsf{I})[x := z]) \longrightarrow (p = x)$$
$$\forall xp.(\exists y \exists z R_\circ yzx \wedge (p = y) \wedge Ix[x := z]) \longrightarrow (p = x)$$
$$\forall xp.(\exists y \exists z R_\circ yzx \wedge (p = y) \wedge Iz) \longrightarrow (p = x)$$
$$\forall xp.(\exists z R_\circ pzx \wedge Iz) \longrightarrow (p = x)$$

Abbreviations (in the style of Restall)

Structural rules:

$$\frac{\Sigma[\Gamma] \vdash A}{\Sigma[\Gamma'] \vdash A} \qquad \equiv \qquad \frac{\Gamma}{\Gamma'}$$

Implicit universals:

$$\forall x, y, z.Rxyz \qquad \equiv \qquad Rxyz$$

Implicit existentials, one-place:

$$Rx(T)z \qquad \equiv \qquad \exists y.Rxyz \wedge Ty$$

Implicit existentials, three-place:

$$R_1x(R_2uv)z \qquad \equiv \qquad \exists y.R_1xyz \wedge R_2uvy$$

Structural postulate:	Frame condition:
$\dfrac{p}{p \circ \mathsf{I}}$ (double line)	$R_\circ x(I)y \leftrightarrow x = y$
$\dfrac{p \bullet (q \circ r)}{q \circ ((\mathsf{B} \bullet p) \bullet r)}$ (double line)	$R_\circ q(R_\bullet(R_\bullet(B)p)r)x \leftrightarrow R_\bullet p(R_\circ qr)x$
$\dfrac{(p \circ q) \bullet r}{p \circ ((\mathsf{C} \bullet q) \bullet r)}$ (double line)	$R_\circ p(R_\bullet(R_\bullet(C)q)r)x \leftrightarrow R_\bullet(R_\circ pq)rx$

Soundness and Completeness (Restall theorems 11.20, 11.37):

> $X \vdash A$ is provable in every model $\mathfrak{M} = \langle \mathcal{F}, \models \rangle$ that satisfies the frame conditions iff in every model, $\forall x \in \mathcal{F}, x \models X \rightarrow x \models A$.

NL_{CL} is conservative over the NL fragment

Worry: continuations might introduce unwanted commutativity.

Conservativity: Let $X \vdash A$ be a sequent built only from the ingredients allowed in NL: $/, \bullet, \backslash$. If NL_{CL} is conservative over the NL fragment, then $X \vdash A$ is provable in NL_{CL} iff it is provable in NL.

- Using $\backslash\!\!\backslash L$, $/\!\!/ L$, $\backslash\!\!\backslash R$ or $/\!\!/ R$ introduces a connective that never goes away, so the final sequent will not be relevant.
- That leaves only the structural rule. It introduces a $\circ$ which eventually has to be eliminated, so applications of the structural rule have to come in matched pairs.
- We can use rules like $\backslash L$ and $/L$ to target an abstracted element. But we could have targeted the same element when it was in-situ, so nothing new can be derived.
- The proof proceeds by extending a falsifying NL model to a falsifying NL_{CL} model.

Open question: decidability?

$\mathbf{NL}_\lambda$

First (rough) resemblance: Morrill 1994

Scope-taking implemented by the interaction of

- a non-associative mode (here, $\bullet$);
- an associative mode (here, via C);
- a wrap mode (here, B)

2^d resemblance: Reductions in the $\lambda\mu$-calculus (de Groote 2001b)

μ-left: $N(\mu\alpha.M) \rightsquigarrow \mu\beta.M[(\alpha X) := \beta(NX)]$

μ-right: $(\mu\alpha.M)N \rightsquigarrow \mu\beta.M[(\alpha X) := \beta(XN)]$

$$\frac{N \bullet (M \circ X)}{M \circ ((\mathsf{B} \bullet N) \bullet X)}\ \mathsf{B}$$

$$\frac{(M \circ X) \bullet N}{M \circ ((\mathsf{C} \bullet X) \bullet N)}\ \mathsf{C}$$

- B allows a scope-taking element to hop leftwards
- C allows a scope-taking element to hop (up) rightwards

Connection both with lambda and with continuations; though in the $\lambda\mu$-calculus, continuations are undelimited.

Third resemblance: Embedding λ-terms into Combinatory Logic

Shönfinkel's mapping (Barendregt 1984:152):

$$\langle x \rangle \equiv x \qquad \mathbb{A}(x, x) \equiv \mathrm{I}$$

$$\langle MN \rangle \equiv \langle M \rangle \langle N \rangle \qquad \mathbb{A}(x, M) \equiv \mathrm{K}M \quad (x \text{ not free in } M)$$

$$\langle \lambda x.M \rangle \equiv \mathbb{A}(x, \langle M \rangle) \qquad \mathbb{A}(x, MN) \equiv \mathrm{S}(\mathbb{A}(x, M))(\mathbb{A}(x, N))$$

where $\mathrm{S}xyz = xz(yz)$, $\mathrm{K}xy = x$, and $\mathrm{I}x = x$ as usual. For example,

$$\langle \lambda x \lambda y.yx \rangle = \mathrm{S}(\mathrm{K}(\mathrm{SI}))(\mathrm{S}(\mathrm{KK})\mathrm{I})$$

David Turner adds clauses, more efficient for linear terms:

$$\mathbb{A}(x, MN) \equiv \mathrm{B}M(\mathbb{A}(x, N)) \qquad (x \text{ not free in } M)$$

$$\mathbb{A}(x, MN) \equiv \mathrm{C}(\mathbb{A}(x, M))N \qquad (x \text{ not free in } N)$$

where $\mathrm{B}xyz = x(yz)$ and $\mathrm{C}xyz = xzy$. Now

$$\langle \lambda x \lambda y.yx \rangle = \mathrm{B}(\mathrm{CI})\mathrm{I}$$

Linear combinators into NL$_{CL}$:

Now adapt the mapping for NL$_{CL}$ (still written $\langle\cdot\rangle$).
Since all abstracts are linear, no need to mention S or K.

$$\begin{aligned}
\langle x \rangle &\equiv x \\
\langle p \bullet q \rangle &\equiv \langle p \rangle \bullet \langle q \rangle \\
\langle \lambda x.p \rangle &\equiv \mathbb{A}(x, \langle p \rangle)
\end{aligned}$$

$$\begin{aligned}
\mathbb{A}(x, x) &\equiv \mathrm{I} \\
\mathbb{A}(x, p \bullet q) &\equiv (\mathrm{B} \bullet p) \bullet \mathbb{A}(x, q) && (x \text{ not free in } p) \\
\mathbb{A}(x, p \bullet q) &\equiv (\mathrm{C} \bullet \mathbb{A}(x, p)) \bullet q && (x \text{ not free in } q)
\end{aligned}$$

Derived inference rule:

$$\frac{\Sigma[\Gamma\lceil p \rceil] \vdash A}{\Sigma[p \circ \langle \lambda x.\Gamma\lceil x \rceil \rangle] \vdash A}\ \lambda$$

Details about $\Gamma\lceil p\rceil$ structures

As usual with λ, we pay for conceptual simplicity with some definitional complexity.

$$\Gamma\lceil p\rceil ::= \quad p \quad | \quad \lambda y.\,\Gamma\lceil p\rceil \quad | \quad q \bullet \Gamma\lceil p\rceil \quad | \quad \Gamma\lceil p\rceil \bullet q$$

This λ "abstracts" only over structures built from $\bullet$ and λ.

Allowed: $\dfrac{A}{A \circ \lambda x.x}$ $\quad \dfrac{A \bullet B}{A \circ \lambda x.(x \bullet B)}$ $\quad \dfrac{\lambda x.(x \bullet B)}{B \circ \lambda y \lambda x.(x \bullet y)}$

Disallowed: $\dfrac{A \circ B}{A \circ \lambda x.(x \circ B)}$ $\quad \dfrac{\lambda x.(x \bullet B)}{B \circ \lambda x \lambda y.(x \bullet y)}$

Crucially linear: x fresh (distinct from every other symbol in Γ).

Example derivation of *John saw everyone*

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{c}\vdots\\ \mathrm{DP} \bullet ((\mathrm{DP}\backslash \mathrm{S})/\mathrm{DP} \bullet \mathrm{DP}) \vdash \mathrm{S}\end{array}}{\mathit{John} \bullet (\mathit{saw} \bullet \mathrm{DP}) \vdash \mathrm{S}}\,\mathrm{LEX}}{\mathrm{DP} \circ \lambda x(\mathit{John} \bullet (\mathit{saw} \bullet x)) \vdash \mathrm{S}}\,\lambda}{\lambda x(\mathit{John} \bullet (\mathit{saw} \bullet x)) \vdash \mathrm{DP}\backslash\!\!\backslash \mathrm{S}}\,\backslash\!\!\backslash R \qquad \mathrm{S} \vdash \mathrm{S}}{\mathrm{S}/\!\!/(\mathrm{DP}\backslash\!\!\backslash \mathrm{S}) \circ \lambda x(\mathit{John} \bullet (\mathit{saw} \bullet x)) \vdash \mathrm{S}}\,/\!\!/L}{\mathit{John} \bullet (\mathit{saw} \bullet \mathrm{S}/\!\!/(\mathrm{DP}\backslash\!\!\backslash \mathrm{S})) \vdash \mathrm{S}}\,\lambda$$

$$\mathbf{everyone}(\lambda x.\mathbf{saw}\, x\, \mathbf{j})$$

Examples, comparisons, interpretations

Compared to Barker and Shan 2006

- Like B&S, gives delimited continuations in a TLG setting
- Like B&S, uses a right identity (Restall's Push and Pop)
- Unlike B&S, NL_{CL} generalizes to nested contexts
- Unlike B&S, no explicit control over evaluation order (yet!)

Compared to Kiselyov and Shan 2007

- Like K&S, gives delimited continuations in an intuitionistic substructural logic
- Unlike K&S, describes an ambiguous language
  - K&S provide explicit reset operator to delimit scope
  - Here, in-situ scope-takers are self-limiting, e.g., a scope-taker like *everyone*: $\mathrm{S} /\!\!/ (\mathrm{DP} \backslash\!\!\backslash \mathrm{S})$ takes scope over any containing S constituent (easily constrained by standard techniques for regulating access to resources, see Barker and Shan for details on scope islands).

Generalizing to nested contexts (contexts inside contexts)

$$\dfrac{\dfrac{\dfrac{\dfrac{p \bullet q \vdash A}{(p \circ \mathsf{I}) \bullet q \vdash A}\,\mathsf{I}}{p \circ ((\mathsf{C} \bullet \mathsf{I}) \bullet q) \vdash A}\,\mathsf{C}}{p \circ ((\mathsf{C} \bullet \mathsf{I}) \bullet (q \bullet \mathsf{I})) \vdash A}\,\mathsf{I}}{p \circ (q \circ ((\mathsf{C} \bullet (\mathsf{C} \bullet \mathsf{I})) \bullet \mathsf{I})) \vdash A}\,\mathsf{C}$$

$$\dfrac{\dfrac{p \bullet q \vdash A}{p \circ \lambda x(x \bullet q) \vdash A}\,\lambda}{p \circ (q \circ \lambda y \lambda x(x \bullet y)) \vdash A}\,\lambda$$

- $\lambda x(x \bullet q)$ is a context: p's delimited continuation relative to the proof of A.
- $\lambda y \lambda x(x \bullet y)$ is a context inside a context: the delimited continuation of q relative to the context $\lambda x(x \bullet q)$.

Example: *Everyone said he left*:

$$he \qquad \lambda R \lambda x. Rxx : (\mathrm{DP} \backslash\!\backslash \mathrm{S}) /\!/ (\mathrm{DP} \backslash\!\backslash (\mathrm{DP} \backslash\!\backslash \mathrm{S}))$$

$$\mathrm{DP} \bullet (\textit{said} \bullet (\mathrm{DP} \bullet \textit{left})) \vdash \mathrm{S}$$

$$\lambda$$

$$\mathrm{DP} \circ \lambda x (x \bullet (\textit{said} \bullet (\mathrm{DP} \bullet \textit{left}))) \vdash \mathrm{S}$$

$$\backslash\!\backslash R$$

$$\lambda x (x \bullet (\textit{said} \bullet (\mathrm{DP} \bullet \textit{left}))) \vdash \mathrm{DP} \backslash\!\backslash \mathrm{S}$$

$$\lambda$$

$$\mathrm{DP} \circ \lambda y \lambda x (x \bullet (\textit{said} \bullet (y \bullet \textit{left}))) \vdash \mathrm{DP} \backslash\!\backslash \mathrm{S}$$

$$\backslash\!\backslash R$$

$$\lambda y \lambda x (x \bullet (\textit{said} \bullet (y \bullet \textit{left}))) \vdash \mathrm{DP} \backslash\!\backslash (\mathrm{DP} \backslash\!\backslash \mathrm{S}) \qquad \mathrm{DP} \backslash\!\backslash \mathrm{S} \vdash \mathrm{DP} \backslash\!\backslash \mathrm{S}$$

$$/\!/ L$$

$$(\mathrm{DP} \backslash\!\backslash \mathrm{S}) /\!/ (\mathrm{DP} \backslash\!\backslash (\mathrm{DP} \backslash\!\backslash \mathrm{S})) \circ \lambda y \lambda x (x \bullet (\textit{said} \bullet (y \bullet \textit{left}))) \vdash \mathrm{DP} \backslash\!\backslash \mathrm{S}$$

$$\mathrm{LEX}$$

$$he \circ \lambda y \lambda x (x \bullet (\textit{said} \bullet (y \bullet \textit{left}))) \vdash \mathrm{DP} \backslash\!\backslash \mathrm{S}$$

$$\lambda$$

$$\lambda x (x \bullet (\textit{said} \bullet (he \bullet \textit{left}))) \vdash \mathrm{DP} \backslash\!\backslash \mathrm{S} \qquad \mathrm{S} \vdash \mathrm{S}$$

$$/\!/ L$$

$$\mathrm{S} /\!/ (\mathrm{DP} \backslash\!\backslash \mathrm{S}) \circ \lambda x (x \bullet (\textit{said} \bullet (he \bullet \textit{left}))) \vdash \mathrm{S}$$

$$\mathrm{LEX}$$

$$everyone \circ \lambda x (x \bullet (\textit{said} \bullet (he \bullet \textit{left}))) \vdash \mathrm{S}$$

$$\lambda$$

$$everyone \bullet (\textit{said} \bullet (he \bullet \textit{left})) \vdash \mathrm{S}$$

$$\mathbf{everyone}((\lambda R \lambda x. Rxx)(\lambda y \lambda x.\mathbf{said}(\mathbf{left}\, x)\, y)) = \mathbf{eo}(\lambda z.\mathbf{said}(\mathbf{lft}\, z)\, z)$$

(cf. Morrill, Fadda & Valentín 2007:52 : $\lambda R \lambda x. Rxx : ((\mathrm{S} \uparrow \mathrm{DP}) \uparrow \mathrm{DP}) \downarrow (\mathrm{S} \uparrow \mathrm{DP}))$

- Dowty 2007: duplication in lexicon, so no contraction in logic
- Unlike Dowty, pronoun not restricted to scoping over VP-oids
- Weak crossover currently unexplained (see Barker and Shan 2006)

Example: *Everyone read the same book*:

$$\vdots$$

$$\textsc{dp} \bullet (\textit{read} \bullet (\textit{the} \bullet (\mathrm{N/N} \bullet \textit{book}))) \vdash \textsc{s}$$

$$\overline{\textsc{dp} \circ \lambda x(x \bullet (\textit{read} \bullet (\textit{the} \bullet (\mathrm{N/N} \bullet \textit{book})))) \vdash \textsc{s}}\ \lambda$$

$$\overline{\lambda x(x \bullet (\textit{read} \bullet (\textit{the} \bullet (\mathrm{N/N} \bullet \textit{book})))) \vdash \textsc{dp}\backslash\!\!\backslash\textsc{s}}\ \backslash\!\!\backslash R$$

$$\overline{\mathrm{N/N} \circ \lambda y \lambda x(x \bullet (\textit{read} \bullet (\textit{the} \bullet (y \bullet \textit{book})))) \vdash \textsc{dp}\backslash\!\!\backslash\textsc{s}}\ \lambda$$

$$\overline{\lambda y \lambda x(x \bullet (\textit{read} \bullet (\textit{the} \bullet (y \bullet \textit{book})))) \vdash (\mathrm{N/N})\backslash\!\!\backslash(\textsc{dp}\backslash\!\!\backslash\textsc{s})}\ \backslash\!\!\backslash R \qquad \textsc{dp}\backslash\!\!\backslash\textsc{s} \vdash \textsc{dp}\backslash\!\!\backslash\textsc{s}$$

$$\overline{(\textsc{dp}\backslash\!\!\backslash\textsc{s})/\!\!/((\mathrm{N/N})\backslash\!\!\backslash(\textsc{dp}\backslash\!\!\backslash\textsc{s})) \circ \lambda y \lambda x(x \bullet (\textit{read} \bullet (\textit{the} \bullet (y \bullet \textit{book})))) \vdash \textsc{dp}\backslash\!\!\backslash\textsc{s}}\ /\!\!/L$$

$$\overline{\lambda x(x \bullet (\textit{read} \bullet (\textit{the} \bullet ((\textsc{dp}\backslash\!\!\backslash\textsc{s})/\!\!/((\mathrm{N/N})\backslash\!\!\backslash(\textsc{dp}\backslash\!\!\backslash\textsc{s})) \bullet \textit{book})))) \vdash \textsc{dp}\backslash\!\!\backslash\textsc{s}}\ \lambda \qquad \textsc{s} \vdash \textsc{s}$$

$$\overline{S/\!\!/(\textsc{dp}\backslash\!\!\backslash\textsc{s}) \circ \lambda x(x \bullet (\textit{read} \bullet (\textit{the} \bullet ((\textsc{dp}\backslash\!\!\backslash\textsc{s})/\!\!/((\mathrm{N/N})\backslash\!\!\backslash(\textsc{dp}\backslash\!\!\backslash\textsc{s})) \bullet \textit{book})))) \vdash \textsc{s}}\ /\!\!/L$$

$$\overline{S/\!\!/(\textsc{dp}\backslash\!\!\backslash\textsc{s}) \bullet (\textit{read} \bullet (\textit{the} \bullet ((\textsc{dp}\backslash\!\!\backslash\textsc{s})/\!\!/((\mathrm{N/N})\backslash\!\!\backslash(\textsc{dp}\backslash\!\!\backslash\textsc{s})) \bullet \textit{book}))) \vdash \textsc{s}}\ \lambda$$

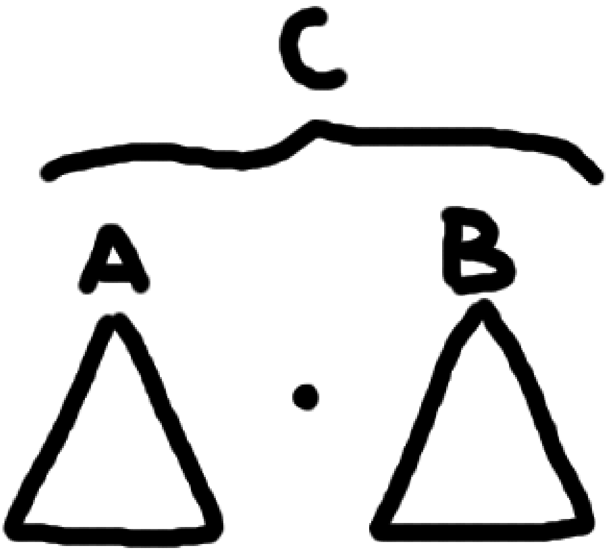
$$\overline{\textit{everyone} \bullet (\textit{read} \bullet (\textit{the} \bullet (\textit{same} \bullet \textit{book}))) \vdash \textsc{s}}\ \textsc{lex}$$

$$\mathbf{everyone}(\mathbf{same}(\lambda f(\lambda y.\mathbf{read}(\mathbf{the}(f(\mathbf{book}))))))$$

(Barker in press: parasitic scope: *same* scopes over *everyone*'s scope)

• as concatenation mode

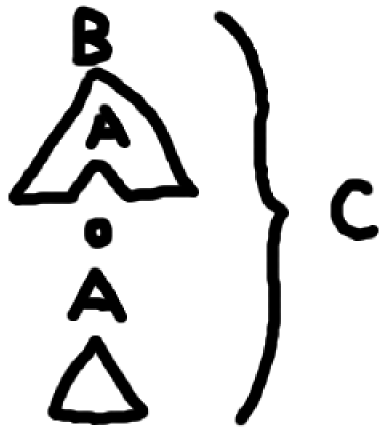
$$A \vdash C/B \qquad (A \bullet B) \vdash C \qquad B \vdash A\backslash C$$



horizontal mode

$\circ$ as plug mode (delimited continuations)

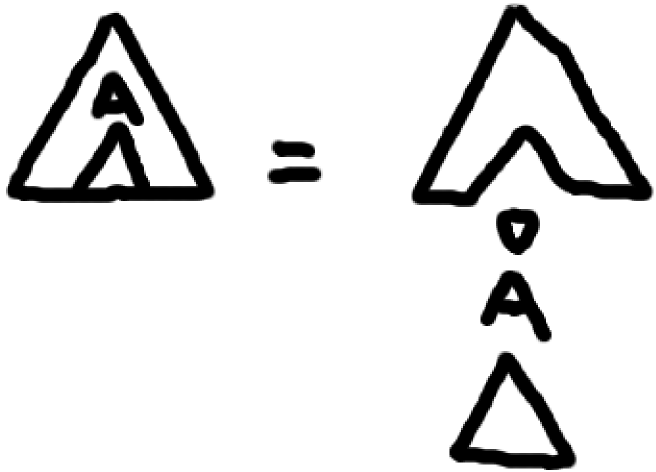
$$A \vdash C /\!\!/ B \qquad (A \circ B) \vdash C \qquad B \vdash A \backslash\!\!\backslash C$$



- $A \backslash\!\!\backslash C$: a C missing an A somewhere inside (Morrill: $C \uparrow A$)
- $A \circ B$: An A plugged into the hole in B.
- $C /\!\!/ B$: a C missing a B at its top. (Morrill: $B \downarrow C$)

$A \backslash\!\!\backslash C$ is a delimited continuation.

Structural equivalence



$$\Gamma[A] = A \circ \Gamma[\,]$$

Delimited, not undelimited continuations

- Logically, undelimited continuations have type $A \rightarrow \bot$
- Computationally, functions that never return
- A strategy: Barker 2002, de Groote 2001b, Bernardi and Moortgat 2007: identify $\bot$ with some useful result type, such as S.
- Limitation: special steps needed to allow a scope-taking element to change the result type of the expression it takes scope over.
- Logically, delimited continuations have type $A \backslash\!\!\backslash B$
- Computationally, delimited continuations are composable
- Conjecture (Barker 2004): natural language makes use of only delimited continuations
  - In-situ *wh*: $\mathrm{Q} /\!\!/ (\mathrm{DP} \backslash\!\!\backslash \mathrm{S})$: *John saw who?*
  - Focus particles (Barker 2004)
  - Pied-Piping (Moortgat circa 2000) *which*: $Rel /\!\!/ (\mathrm{DP} \backslash\!\!\backslash \mathrm{DP})$: *the book [the author of which] Alice admires*

ACGs

- Allow λ to abstract over $\circ$ as well as over $\bullet$ by adding two postulates. This makes the $\bullet$ mode and the $\circ$ mode symmetric.
- Let SAW abbreviate $\lambda y \lambda x(y \bullet (\mathit{saw} \bullet x))$
- Let EVERYONE abbreviate $\lambda \kappa(\mathit{everyone} \circ \kappa)$
- Remember, "$x \circ f$" is value $\circ$ context, argument $\circ$ functor.

$$\dfrac{\dfrac{\dfrac{\dfrac{\vdots}{\mathit{John} \bullet (\mathit{saw} \bullet \mathit{everyone}) \vdash \text{S}}}{\mathit{everyone} \circ \lambda x(\mathit{John} \bullet (\mathit{saw} \bullet x)) \vdash \text{S}}\ \lambda^{\bullet}}{\mathit{everyone} \circ (\mathit{John} \circ \lambda y \lambda x(y \bullet (\mathit{saw} \bullet x))) \vdash \text{S}}\ \lambda^{\bullet}}{\dfrac{(\mathit{John} \circ \lambda y \lambda x(y \bullet (\mathit{saw} \bullet x))) \circ \lambda \kappa(\mathit{everyone} \circ \kappa) \vdash \text{S}}{(\mathit{John} \circ \text{SAW}) \circ \text{EVERYONE} \vdash \text{S}}\ \text{ACG-ABBREV}}\ \lambda^{\circ}$$

So $\bullet$ corresponds to phenogrammar, $\circ$ corresponds to techtogrammar, and the abbreviations characterize the relationship between the levels.

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