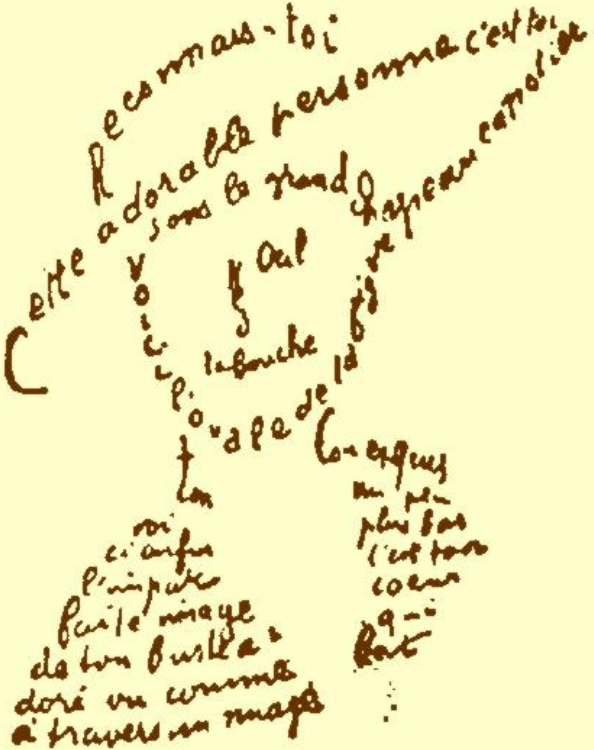
Reconnais-toi
Cette adorable personne c'est toi
sous la grand chapeau carolingien
Voici
Oui
Voici la bouche
Voici le pistolet de la
Ton
Quelques
un pli-
plus bas
c'est ton
coeur
qui
a enfin
l'impact
Suite image
de ton Buste
doré en comme
à travers un nuage

Yet Another Dynamic Logic

Philippe de Groote
LORIA & Inria-Lorraine

A Type-theoretic reconstruction of DRT

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Proposed solution:

- to interpret a sentence according to both its left and right contexts;
- to abstract these two kinds of contexts over the meaning of the sentences.

Typing the left and the right contexts

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Montague semantics is based on Church's simple type theory, which provides a full hierarchy of functional types built upon two atomic types:

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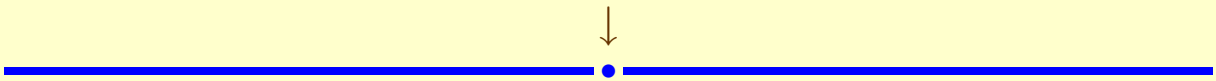

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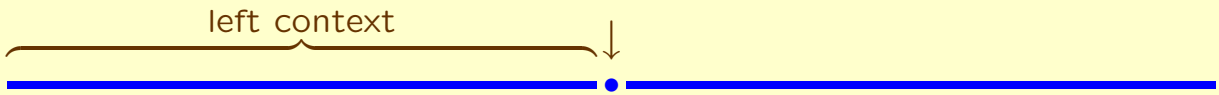

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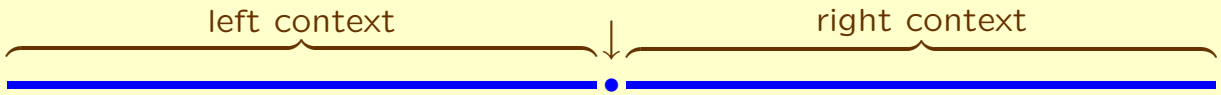

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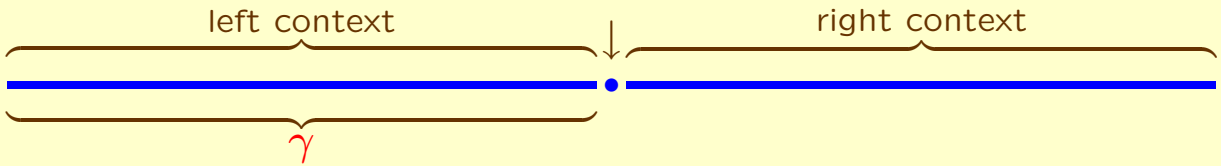

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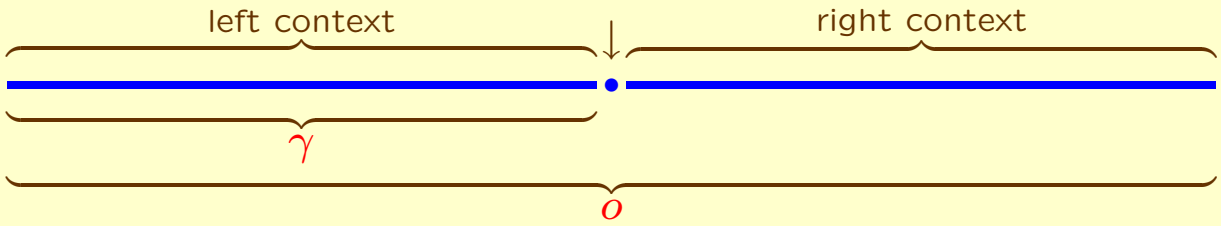

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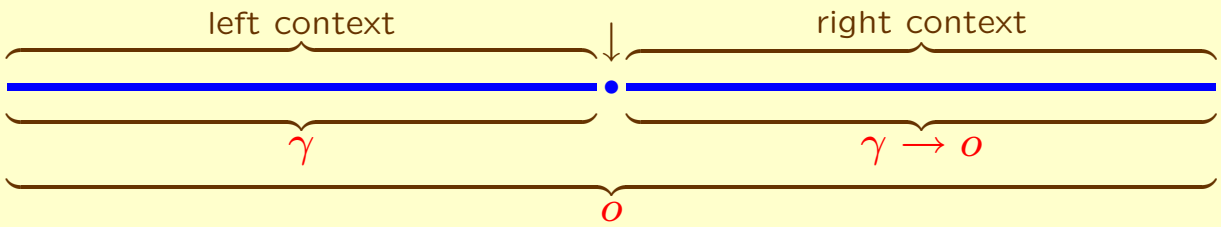

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Composition of two sentence interpretations

$$[\![S_1 . S_2]\!] = \lambda e \phi . [\![S_1]\!]\, e\, (\lambda e' . [\![S_2]\!]\, e'\, \phi)$$

Semantic interpretation of the syntactic categories

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Montague's interpretation

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This interpretation results in handcrafted lexical semantics such as the following:

$$[\![\text{every}]\!] = \lambda n \psi e \phi.\, (\forall x.\, \neg(n\, x\, e\, (\lambda e.\, \neg(\psi\, x\, (x{::}e)\, (\lambda e.\, \top))))) \wedge \phi\, e$$

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Questions:

- is there a systematic way of obtaining the new lexical semantics from Montague's ?
- can we find any "modular" presentation of the approach ?
- is there some dynamic logic hidden in the approach ?

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Consequently, our logic will be based on conjunction and existential quantification (defined as primitives). The other connectives will be obtained using negation (a third primitive) and de Morgan's laws.

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Negation

Negation cannot be canonically defined because we do not want the continuation of the discourse to fall into the scope of the negation:

$$\sim A \triangleq \lambda e \phi.\, \neg\, (A\, e\, (\lambda e.\, \top)) \wedge \phi\, e$$

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$$\begin{aligned} \overline{R_i^n\, t_1\, \ldots\, t_n} &= \lambda e \phi.\, R_i^n\, t_1\, \ldots\, t_n \wedge \phi\, e \\ \overline{\neg A} &= \sim \overline{A} \\ \overline{A \wedge B} &= \overline{A} \sqcap \overline{B} \\ \overline{\exists x.\, A} &= \Sigma x.\, \overline{A} \end{aligned}$$

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This embedding is such that, for every term e of type γ:

$$A \equiv \overline{A}\, e\, (\lambda e.\, \top)$$

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We then define two translations of the (first-order) relational symbols:

$$\overline{R_i^n} \triangleq \lambda x_1 \ldots x_n e \phi.\, R_i^n\, x_1\, \ldots\, x_n \wedge \phi\, e$$
$$\overline{\overline{R_i^n}} \triangleq \lambda x_1 \ldots x_n e \phi.\, R_i^n\, x_1\, \ldots\, x_n \wedge \phi\, (x_1 :: \cdots :: x_n :: e)$$

Donkey sentence revisited

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Montague-like semantic interpretation:

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[\![\text{donkey}]\!] &= \mathbf{donkey} \\
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With the dynamic interpretation we have that:

$$[\![\text{beats}]\!]\,[\![\text{it}]\!]\,([\![\text{every}]\!]\,([\![\text{who}]\!]\,([\![\text{owns}]\!]\,([\![\text{a}]\!]\,[\![\text{donkey}]\!]))\,[\![\text{farmer}]\!]))$$

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β-reduces to the following term (modulo de Morgan's laws):

$$\lambda e\phi.\,(\forall x.\,\mathbf{farmer}\,x\;\supset\;(\forall y.\,\mathbf{donkey}\,y\;\supset\;(\mathbf{own}\,x\,y\;\supset\;\mathbf{beat}\,x\,(\mathsf{sel}\,(x{::}y{::}e)))))\;\wedge\;\phi\,e$$